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## *Survey paper on transform techniques & its applications*

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## **Abstract:**

In the realm of image processing, selecting the appropriate transform and their importance. This survey paper produce a comparative analysis of the Curvelet Transform alongside other transforms, exploring their efficacy in various image processing techniques. Specifically, it delves into the applications of the Curvelet Transform in image compression, phase recognition, and denoising, with the objective of achieving higher compression rates while maintaining quality reconstruction. The introduction of wavelets has significantly transformed compression techniques, adding a new dimension to the process. However, the Curvelet Transform demonstrates superior performance, particularly in terms of Peak Signal-to-Noise Ratio (PSNR). Face recognition emerges as a crucial aspect across numerous applications, including video surveillance, criminal investigations, forensic analysis, secure electronic banking, mobile phones, credit cards, and secure building access. The distinguishing feature of the Curvelet Transform lies in its multi-scale directional nature, allowing for an almost optimal non-adaptive sparse representation of objects with edges. Moreover, its utility extends beyond traditional image processing applications, proving effective in diverse fields.

## **Keywords:**

Image processing, Curve let transform, Wavelet Transform.

## 1. Introduction:

In the realm of image processing, diverse methodologies are used to analyze images in various domains including denoising, compression, face recognition, biomedical applications, and beyond. These analyses often leverage different types of transforms such as the Fourier Transform, Wavelet Transform, and Curvelet Transform. Presently, both Wavelet and Curvelet transforms are widely adopted across all areas of image processing.

### 1.1. Digital image processing system:

A typical digital image processing system comprises several key components: image segmentation, feature extraction, pattern recognition, thresholding, and error classification. The primary objective of image processing is to extract essential information from the image. This involves condensing the image to certain defining characteristics, and analyzing these characteristics yields the relevant information.

The process flow diagram of a typical digital image processing system illustrates the sequence of operations involved in these tasks, guiding the overall workflow.

### 1.2. Wavelet transform:

In signal processing, while we can obtain a time-amplitude representation of signals, this often falls short in providing a proper representation. To achieve a more comprehensive representation, we turn to frequency domain representation. However, the Fourier transform, while valuable, lacks the ability to provide time information.

To address this limitation, Wavelet transforms come into play. Wavelets are functions defined over a finite interval with an average value of zero. The fundamental concept behind the wavelet transform is to represent any arbitrary function (t) as a combination of these wavelets or basic functions. These basic functions, or "baby wavelets," are derived from a single prototype wavelet known as the mother wavelet through dilation and translation operations.

The Discrete Wavelet Transform (DWT) of a finite-length signal  $s(n)$  comprising  $N$  components is expressed by an  $N \times N$  matrix, particularly for image processing applications.

### 1.3. Curvelet transform:

To address the limitations of the Wavelet Transform, the Curvelet Transform was developed. The Curvelet Transform is a highly effective model that not only considers multi-scale time-frequency local aspects but also incorporates the direction of features. Initially developed by

Candes and Donoho in 1999, it offers two types: the unequally spaced Fast Fourier Transform and the wrapping-based fast Curvelet Transform.

In the Curvelet Transform, the relationship between width and length is governed by anisotropic scaling, typically represented as  $\text{Width} \sim \text{Length}^2$ , known as parabolic scaling. Unlike wavelets, where elements are indexed solely by scale and location, Curvelets are indexed by scale, location, and orientation parameters. This enables a more precise representation of features.

The Curvelet Transform can be applied in both continuous and digital domains. In the frequency domain, angle polar wedges or angle trapezoid windows are utilized. Initially, the construction of the Curvelet Transform was redesigned as the fast discrete Curvelet Transform (FDCT) by Candes in 2006, aiming for simplicity and ease of use.

The FDCT serves as the second generation Curvelet Transform and can efficiently implement the Discrete Curvelet Transform (DCT). At a given scale and orientation, both the image and the Curvelet are transformed into the Fourier domain. The product of the Curvelet and the image is then obtained in the Fourier domain. The Inverse Fast Fourier Transform (IFFT) is applied to this product to obtain a set of Curvelet coefficients. To perform the Inverse Fourier Transform (IFT), the trapezoidal wedge obtained from the frequency response of a Curvelet is wrapped into a rectangular support. The spectrum inside the wedge is periodically tilted, allowing the rectangular region to collect the fragmented portions of the wedge through periodic tilting.

## **2. Importance of curvelets over wavelets:**

Curvelets exhibit superiority over wavelets in the following scenarios:

- i) Curvelets provide an optimally sparse representation of objects with edges.
- ii) Curvelets offer optimal image reconstruction in severely ill-posed problems.
- iii) Curvelets provide an optimal sparse representation of wave propagators [3].

### **2.1. Continuous curvelet transform:**

The Continuous Curvelet Transform has undergone two significant revisions. The initial version, often denoted as the "Curvelet '99" transform, employed a complex series of steps that included ridgelet analysis of the radon transform of an image. However, its performance was notably slow. Subsequently, in 2003, the algorithm was modified. The Ridgelet Transform was

abandoned, resulting in a reduction of redundancy within the transform and a substantial increase in speed.

**2.2 In a heuristic argument is made that all curvelets fall into one of three categories:**

- i) A curvelet whose length-wise support does not intersect a discontinuity will result in a curvelet coefficient magnitude of zero (Fig. 1.a).
- ii) A curvelet whose length-wise support intersects with a discontinuity, but not at its critical angle, will yield a curvelet coefficient magnitude close to zero (Fig. 1.b).
- iii) A curvelet whose length-wise support intersects with a discontinuity and is tangent to that discontinuity will produce a curvelet coefficient magnitude much larger than zero (Fig. 1.c) [2].

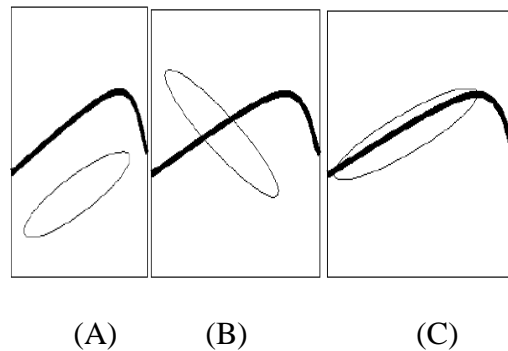


Figure. 1: Curvelet Type A, Curvelet Type B and Curvelet Type C

**3. Description of curvelet transform:**

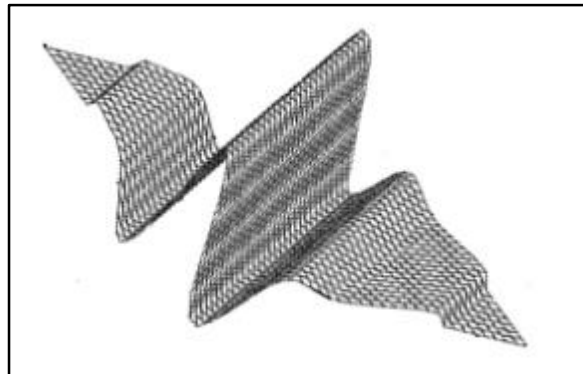
Essentially, the curvelet transform extends the ridgelet transform to enable multiple scale analysis. To comprehend this, let's begin with the definition of the ridgelet transform. Given an image function  $f(x, y)$ , the continuous ridgelet transform is expressed as:

$$\mathfrak{R}_f(a, b, \theta) = \iint \psi_{a, b, \theta}(x, y) f(x, y) dx dy$$

Where  $a > 0$  is the scale,  $b \in \mathbb{R}$  is the translation and  $\theta \in [0, 2\pi]$  is the orientation. The ridgelet is defined as:

$$\psi_{a, b, \theta}(x, y) = a^{-\frac{1}{2}} \psi\left(\frac{x \cos \theta + y \sin \theta - b}{a}\right)$$

Fig. 2 depicts a typical ridgelet [11]. It is oriented at an angle  $\theta$ , and is constant along lines:  $x \cdot \cos\theta + y \cdot \sin\theta = \text{const}$ . It can be seen that a ridgelet is linear in edge direction and is much sharper than a conventional sinusoid wavelet.



*Figure. 2: A ridgelet waveform*

For comparison, the 2-D wavelet is given as:

$$\psi_{a_1, a_2, b_1, b_2}(x, y) = a_1^{-\frac{1}{2}} a_2^{-\frac{1}{2}} \psi\left(\frac{x - b_1}{a_1}\right) \psi\left(\frac{y - b_2}{a_2}\right)$$

As observed, the ridgelet shares similarities with the 2-D wavelet, except that the point parameters  $(b_1, b_2)$  are replaced by the line parameters  $(b, \theta)$ . In other words, the two transforms are related by:

Wavelet:  $\Psi_{\text{scale, point-position}}$

Ridgelet:  $\Psi_{\text{scale, line-position}}$

This implies that ridgelets can be adjusted to various orientations and scales to generate curvelets. Figure 3 illustrates a single curvelet and curvelets tuned to two scales with different numbers of orientations at each scale. Curvelets offer complete coverage of the spectrum in the frequency domain, signifying that there is no loss of information in the curvelet transform concerning capturing frequency information from images. Figure 4 depicts the curvelet tiling and spectrum coverage of a 512x512 image with 5 scales [4]. The shaded wedge illustrates the frequency response of a curvelet at orientation 4 and scale 4. Notably, the spectrum covered by curvelets is comprehensive. In contrast, there are numerous gaps in the frequency plane of Gabor filters (bottom of Fig. 3).

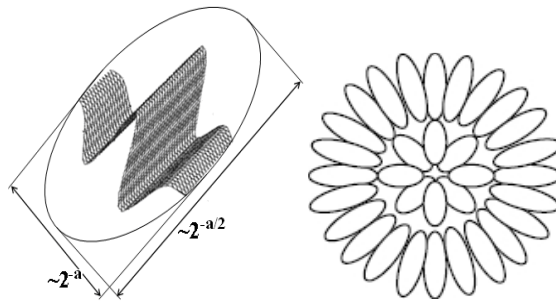


Figure. 3: a single curvelet. & curvelets tuned to different scales and orientations

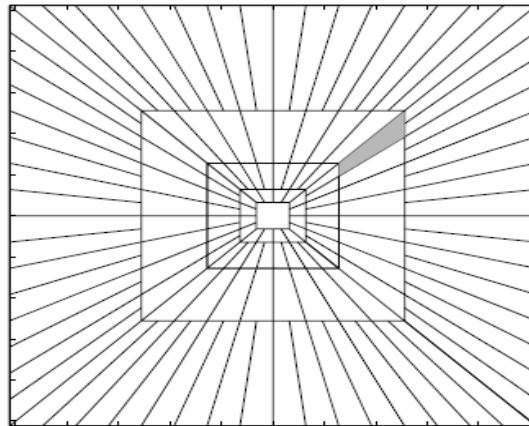


Figure. 4: the tiling of frequency plan by curvelets [9]

#### 4. Application of curvelet transform:

In this section, we will explore the applications of curvelets in various fields such as image processing, fluid mechanics, seismic exploration, solving of PDEs, and compressed sensing. This review aims to demonstrate their potential as a viable alternative to wavelet transforms in certain scenarios.

##### 4.1. Image processing:

In 2002, the initial application of the first-generation curvelet transform for image denoising was conducted by Starck et al. [7] and by Candès and Guo [8]. Subsequently, the utilization of first-generation curvelets expanded to tasks such as image contrast enhancement [11] and representation of astronomical images in 2003, and fusion of satellite images in 2005. Following the introduction of the effective second-generation curvelet transform [12] in 2004, the adoption of curvelets surged across various fields encompassing image/video processing, denoising, and classification.

In the first model [13], a curvelet shrinkage technique is applied to the noisy data, followed by further processing with projected total variation diffusion to suppress pseudo-Gibbs artifacts. In the second model [14], curvelet-based methods are employed to preserve edges and textures effectively.

#### **4.2. Compressed sensing:**

A noteworthy application of the curvelet transform is in compressed sensing (CS), also known as compressive sampling [6], which addresses the challenge of inverse problems with highly incomplete measurements. CS introduces a novel sampling paradigm that combines imaging and compression simultaneously. According to CS theory, a compressible unknown signal can be reconstructed using a small number of random measurements through scarcity-promoting nonlinear recovery algorithms. This requirement for measurements is significantly fewer than those needed by traditional methods adhering to the Shannon/Nyquist sampling theorem, where the sampling rate must be at least twice the maximum frequency of the signal. CS-based data acquisition relies on the sparsity of the signal rather than its bandwidth. CS holds potential for revolutionizing the design of measurement devices across various engineering domains such as medical magnetic resonance imaging (MRI) and remote sensing. This is particularly beneficial for scenarios involving incomplete or inaccurate measurements constrained by physical limitations or costly data acquisition processes.

#### **4.3. Turbulence analysis in fluid mechanics**

Curvelets have found application in investigating the non-local geometry of eddy structures and extracting the coherent vortex field in turbulent flows. This utilization of curvelets marks their emergence in the field of turbulence analysis, with the potential to surpass the wavelet representation of turbulent flows discussed in [6]. Leveraging the multi-scale geometric properties facilitated by curvelets, researchers can examine the evolution of structures associated with the primary ranges of scales defined in Fourier space. Moreover, curvelets maintain localization in physical space, enabling a geometrical study of these structures.

#### **5. Conclusion:**

This paper provides a comprehensive review of the Curvelet transform and its various applications. With ongoing research and broadening scopes, Curvelets have the potential to be applied across diverse fields, leading to revolutionary outcomes. This underscores the suitability of Curvelet Transforms for representing singularities over geometric structures in



images compared to Wavelet counterparts. Curvelets are specifically designed to handle data representing singularities on curves, whereas Wavelets are more effective for point singularities. Thus, Curvelet transform offers distinct advantages in processing image data.

## 6. Future scope:

- The computational cost of curvelets exceeds that of wavelets, prompting research into fast curvelet algorithms to reduce processing time. The exploration of Curvelets in 3D has emerged as a promising research avenue.
- Presently, curvelets are constructed in the Fourier domain, lacking an explicit space-domain formulation. This poses challenges in various applications, including numerical modeling of PDEs. Developing a space-domain formulation for curvelets remains a significant challenge.
- Investigating rotation and scale invariance aims to enhance curvelet retrieval performance. Additionally, the application of curvelet features in color image retrieval and semantic learning will be explored.
- Exploring suitable thresholding functions that leverage the unique characteristics of the curvelet transform is crucial. This aspect is particularly significant for curvelet applications involving edge detection, denoising, and numerical simulation.
- The potential application of curvelet transform in criminal investigation and forensic analysis is an emerging area of research. This includes image recognition from various angles of the body and the analysis of body motion, which presents promising avenues for further exploration.

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