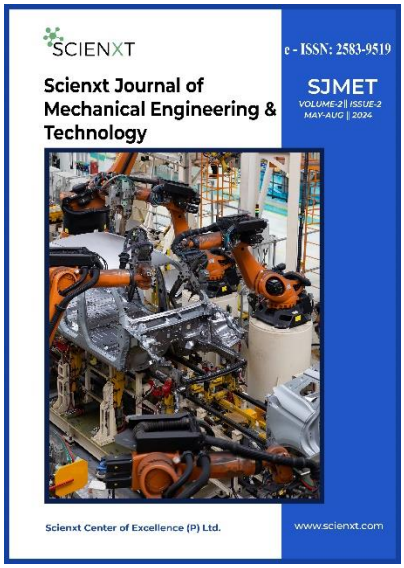


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# *Exact solutions of an MHD layer elastic o-viscous fluid flow of second order type with angle of inclination on the porous boundary*

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## Abstract:

Exact solution of an in-compressible fluid of second order type by causing disturbances in the liquid which is initially at rest due to bottom oscillating sinusoidally has been obtained in this paper. The results presented are in terms of non-dimensional elastic- viscosity parameter ($\beta$) which depends on the non-Newtonian coefficient and the frequency of excitation ($\sigma$) of the external disturbance while considering the magnetic parameter (m), angle of inclination ($\theta$) and porosity (k) of the medium into account. The flow parameters are found to be identical with that of Newtonian case as $\beta \to 0$, $\theta \to 0$, $m \to 0$ and $k \to \infty$.

## Keywords:

Elastico viscous fluid, second order fluid, electico viscous parameter, porous media, magnetic parameter.

## 1. Introduction:

The study of flow past porous boundaries has several important applications in the fields of science, engineering, technology, bio physics, and astrophysics and space dynamics. Transpiration cooling of reentry vehicles and rocket boosters, cross hatching on the ablative surfaces and film vaporization in combustion chambers are few such applications. In the chemical and nuclear reactors the problem assumes greater significance. Over a period of time, in all chemical reactors, slurry adheres to the walls of the reactor and gets consolidated. As a result of which, the chemical compounds within the reactor percolates through the boundaries causing either loss of production and or consuming more reaction time. In many situations, so as to reduce the reaction time which is a subject of prime importance, the reactor chamber is also subjected to sinusoidal vibrations. Further, due to the presence of charged particles either in the in the chemical or nuclear reactor, induced magnetic effects are generated. In which case, the problem is more complicated. Also, the problem assumes greater importance especially in biological systems where the secretion through glands is involved. Many times, either in the chemical processing units or in biological system, the secreted fluid is not only viscous but is also elastico viscous. The presence of elastico viscous nature of the fluid and the presence of magnetic field, causes drastic effects in evaluating the characteristic features of the fluid flow. This motivated the study and analysis of the problem in greater detail. The knowledge of flow through porous media is useful in the recovery of crude oil efficiently from the pores of reservoir rocks by displacement with immiscible water (vide, Rudraiah et al., 1979). The flow through porous media occurs in the ground-water hydrology, irrigation, drainage problems and also in absorption and filtration processes in chemical engineering. This subject has wide spread applications to specific problems encountered in the civil engineering and agriculture engineering, and many industries. Thus the diffusion and flow of fluids through ceramic materials as bricks and porous earthenware has long been a problem of the ceramic industry. The Scientific treatment of the problem of irrigation, Soil erosion and tile drainage are present developments of porous media. In hydrology, the movement of trace pollutants in water systems can be studied with the knowledge of flow through porous media. The principles of this subject are useful in recovering the water for drinking and irrigation purposes. Thurson (1972) was the earliest to recognize the viscoelastic nature of blood and that the viscoelastic behavior is less prominent with increasing shear rate. Lamb (1932) has discussed the viscous flow over an oscillatory bottom earlier in his treatise on hydrodynamics. Berman (1958) studied the problem of two- dimensional steady state Newtonian laminar flow in a channel with porous

walls. An exact, analytical expression for the dependence of velocity on the pressure gradient has been derived. The response of a Coleman-Noll (1960) Second order visco- elastic fluid occupying a semi- infinite region due to harmonic oscillation of its bottom has been investigated later by Pattabhiramacharyulu (1964, 1966). Vidyanidhi, V. and Nigam S.D. (1967) studied secondary flow in rotating channel. Oscillatory motion of an electrically conducting viscoelastic fluid over a stretching sheet in a saturated porous medium was studied by K. Rajagopal (1984). A visco-elastic effect of non- Newtonian flow through porous media was studied by Gupta R.K. and Sridhar T (1985). Pascal and Pascal (1989) studied the visco elastic effects in non-Newtonian steady flows through porous media. Flow of a visco elastic fluid over a stretching sheet was studied by Rajagopal (2006). Ariel P.D. (1994) studied the flow of a visco elastic fluid past a porous plate. Petrov A.G. (2000) examined analytically the unsteady flow of Bingham fluid caused by a abruptly applied pressure gradient. With respect to the flows of non- Newtonian fluids between two parallel porous walls, Ariel (2002) obtained exact analytical solutions of laminar flow of a second grade visco-elastic fluid employing two geometries. An oscillatory plate temperature effect of free convection flow of dissipative fluid between long vertical parallel plates was studied by Narahari (2009). Singh and Paul (2006) presented an analysis of the transient free convective flow of a viscous incompressible fluid between two parallel vertical walls occurring as a result of asymmetric heating/cooling of the walls. Turkyilmazoglu(2009) examined exact solutions for the incompressible viscous fluid of a porous rotating disk flow. Exact solution corresponding to viscous in compressible Newtonian conducting fluid flow due to a porous rotating disk by M. Turkyilmazoglu(2009). The effect of suction and blowing on purely analytic solutions of the compressible boundary layer flow due to a porous rotating disk with heat transfer studied by Turkyilmazoglu(2009). Later, (2014) had examined the problem of unsteady flow of an incompressible viscous electrically conducting fluid in the tube of elliptical cross section under the influence of the magnetic field. Subsequently, (2015) studied the unsteady flow of an incompressible viscous fluid in a tube of spherical cross section on a porous boundary. Recently, (2015) had examined the problem of unsteady MHD flow of elastico - viscous incompressible fluid through a porous media between two parallel plates under the influence of magnetic field.

The aim of the present paper is to study a class of exact solutions for the flow of incompressible electrically conducting elastic-viscous fluid of second order fluid by taking into account the magnetic field,

The momentum equation of the fluid flowing through a generalized porous medium as suggested by Yamamoto and Yamamoto Iwamura (1974) is given by

$$\rho \frac{dq}{dt} = divS - \frac{\mu}{k} \bar{q}$$

And the continuity equation for incompressible homogeneous fluid

Noll (1958) defined a simple material as a substance for which stress can be determined with entire knowledge of the history of the strain. This is called simple fluid, if it has property that all local states, with the same mass density, are intrinsically equal in response, with all observable differences in response being due to definite differences in the history. For any given history g(s), a retarded history $g_{\psi}(s)$ can be defined as:

$$g_{\psi}(s) = g(\psi s): 0 \le \psi \le 1 \qquad 0 \le s \le \infty. \tag{1}$$

$\psi$ being termed as a retardation factor.

Assuming that the stress is more sensitive to recent deformation that to the deformations at distant past, Coleman and Noll proved that the theory of simple fluids yields the theory of perfect fluids as $\psi \to 0$ and that of Newtonian Fluids as a correction (up to the order of $\psi$ ) to the theory of the perfect fluids. Neglecting all the terms of the order of higher than two

in $\psi$, We have incompressible elastic viscous fluid of second order type whose constitutive relation is governed by:

$$S = -PI + \phi_1 E^{(1)} + \phi_2 E^{(2)} + \phi_3 E^{(1)^2}$$

where,

$$E^1 = U, + U \tag{3}$$

And

$$E^2_{ij} = A, + A_{i,j} + A_{j,i} + 2U_{m,i} U_{m,j} \tag{4}$$

In the above equations, S is the stress-tensor, Ui and Ai are the components of velocity and acceleration in the direction of the $i$ coordinate $X_I$ while $P$ is indeterminate hydrostatic pressure. The coefficients $\phi_1$, $\phi_2$ and $\phi_3$ are material constants. The constitutive relation for general Rivlin – Ericksen (1955) fluid also reduces to equation when the squares and higher

orders of E 2 are neglected, while the coefficients being constants. Also the non-Newtonian models considered by Reiner (1945) could be obtained from equation (2) when $\phi_2 = 0$ and naming $\alpha_3$ as the coefficient of cross viscosity. With reference to the Rivlin - Ericksen fluids $\alpha_3$ may be called as the coefficient of viscosity.

It has been reported that a solution of poly - iso - butylene in cetane behaves as a second order fluid and that Markovitz determined the constants $\alpha_1$ , $\alpha_2$ and $\alpha_3$ In many of the chemical processing industries, slurry adheres to the reactor vessels and gets consolidated. As a resultant of this, the chemical compounds within the reactor vessel percolates through the boundaries causing loss of production and consuming more reaction time. In view of such technological and industrial importance wherein the heat and mass transfer takes place in the chemical industry, the problem by considering the permeability of the bounding surfaces in the reactors attracts the attention of several investigators.

Introducing the following non dimensional variables as:

$$T = \frac{\rho L^2 t}{\mu} \qquad \varphi = \rho L^2 p$$

$$\phi_1 \qquad 2$$

$$P = \frac{\phi_1^2 p}{\mu^2 L^2} \qquad \frac{X_i}{L} = x_i \qquad \frac{Y_i}{L} = y_i$$

$$\phi_3 = \rho L^2 \upsilon_c \qquad A_i = \frac{\phi_1^2 a_i}{\rho^2 L^3} \qquad \theta_0 = \frac{\theta \phi_1}{L^2}$$

$$S_{i,j} = \frac{\phi_1^2 s_{i,j}}{\rho L^2} \qquad E^{(1)}_{i,j} = \frac{\phi_1 e^{(1)}_{i,j}}{\mu L^2} \qquad E^{(2)}_{i,j} = \frac{\phi_1^2 e^{(2)}_{i,j}}{\rho^2 L^4}$$

$$K = \frac{kL^2}{\phi_1} \qquad M = \frac{m\phi_1}{L^2}$$

Where $T$ is the (dimensional) time variable, and $\rho$ the mass density and L a characteristic length.

We consider a class of plane flows given by the velocity components

$u_1 = u\,(y, t)$ and $u_2 = 0$ (5)

in the directions of rectangular Cartesian coordinates x and y. The velocity field given by (5) identically satisfies the incompressibility condition. The stress can now be obtained in the non-dimensional form as:

$$\tau_{xx} = -p + \upsilon\left(\frac{\partial u}{\partial y}\right)^2 \qquad (6)$$

$$\tau_{zz} = -p + (\upsilon + 2\beta)\left(\frac{\partial u}{\partial y}\right)^2 \qquad (7)$$

$$\tau_{xy} = \frac{\partial u}{\partial y} + \beta\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right) \qquad (8)$$

In view of the above, the equations of motion in the present case of porous boundary will yield

$$\frac{\partial u}{\partial t} = \frac{\partial x}{\partial y^2} + \theta\frac{\partial}{\partial t}\left(\frac{\partial}{\partial y^2}\right) - \left(\frac{1}{k} + m\right)u \qquad (9)$$

and

$$0 = -\frac{\partial p}{\partial y} + (2\beta+\upsilon)\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (10)$$

Eqn (9) shows that $\partial x$ must be independent of space variables and hence may be taken as $\xi(t)$ . Eqn (10) now yields

$$p = p_0(t) - \xi(t)x + (\upsilon + 2\beta)\left(\frac{\partial u}{\partial y}\right)^2 \qquad (11)$$

Considering $\xi(t) = 0$ , the flow characterized by the velocity is given by:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \beta\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) - \left(\frac{1}{k} + m + \theta\right)u \qquad (12)$$

Where k is the non-dimensional porosity constant. It may be noted that the presence of  changes the order of differential from two to three.

## 2. Disturbance of a liquid at rest due to the sinusoidal oscillations of the bottom:

The oscillations of a classical viscous liquid on the upper half of the plane $y \geq 0$ with the bottom oscillating with the velocity $\alpha e^{i\sigma t}$ are examined in the present case. The motion of the

second order

fluid is governed by Eqn (12) with boundary conditions

$$u(0, t) = \alpha e^{i\sigma t} \qquad (13)$$

$$u(\infty, t) = 0 \qquad (14)$$

Assuming the trial solution as:

$$u(y, t) = \alpha e^{j\sigma t} f(y) \qquad (15)$$

$$f''(y) = p^2 f(y) \qquad (16)$$

Where

When expressed in polar form

$$\frac{\pi}{4} \quad \frac{\varepsilon}{2} \qquad \frac{\pi}{4} \quad \frac{\varepsilon}{2}$$

(18)

$$\frac{[(\beta\sigma^2 + \frac{1}{k} + m + \theta)^2 + (\sigma - (\frac{\beta}{k} + m\beta + \beta))^2]^{1/2}}{\sqrt{(1+\beta^2\sigma^2)}}$$

$$\varepsilon = \tan^{-1}(Q) \quad \text{and}$$

$$Q = \frac{\beta\sigma^2 + m + \theta}{\sigma - (\frac{\beta}{k} + m\beta + \beta)}$$

Also the conditions satisfied by $f(y)$ are

$$f(0) = 1, f(\infty) = 0 \qquad (19)$$

This yields the solution

$$f(y) = \exp(-yr(\cos(\frac{\pi}{4} - \frac{\varepsilon}{2} \qquad \frac{\pi}{4} \quad \frac{\varepsilon}{2}$$

and hence

$$+ i\sin(\quad 24$$

(20)

The flow is thus represented by standing transverse wave with its amplitude rapidly diminishing with increasing distance from the plane. This phenomenon is independent of $\nu$ as noticed for all two - dimensional flows.

The magnification factor $A$ of the amplitude of this wave, with respect to the amplitude of the disturbance ( ) , may be written as

$$(A^*)^2 = (\text{Real part of } u(y,t))^2 + (\text{Imaginary part } u(y,t))^2 \quad (22)$$

$$\chi y^* = \frac{y\sqrt{r}}{2}\left[\cos\frac{\varepsilon}{2} + \sin\frac{\varepsilon}{2}\right| \quad (24)$$

Where

$$\chi = \frac{1}{(1+\beta^2\sigma^2)^{\frac{1}{4}}}\sqrt{\frac{Q+\sqrt{1+Q^2}}{1+Q^2}} \quad (25)$$

and

$$\chi = \frac{1}{\sqrt{2}}\left[\frac{\mu\sigma}{(1+\beta^2\sigma^2)^2}\right] \quad (26)$$

The flow rate is given by $= \int_0^1 u(y,t)dy$ (27)

The skin friction on the plate is given by

$$\left[\frac{\partial u}{\partial y} \quad \frac{\partial^2 u}{\partial y^2}\right]_{y=0}$$

## 3. Conclusions:

In this paper , a problem in studied in order to show the effect of the magnetic field and angle of inclination on unsteady flow of a visco-elastic fluid flow of second order type creating sinusoidal disturbances under the influence of magnetic field on the porous boundary. As $m$ □ 0 and □□ 0 the results obtained for the velocity field in agreement to that of "Murthy etal (2007)", "Kulkarni (2015)" respectively. The case of Newtonian fluid can be realized as $\beta \to 0$, $k \to \infty$, $m \to 0$ and $\theta \to 0$ .

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