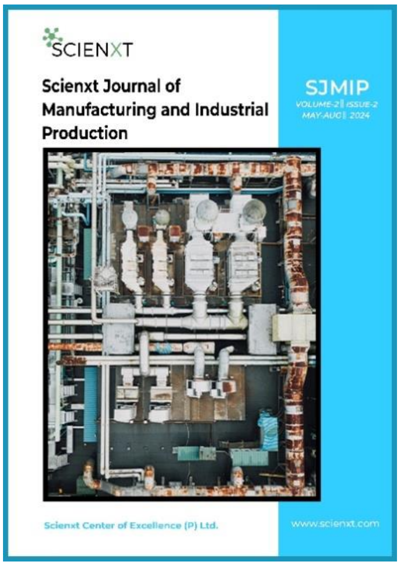

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The study of the visco-elastic and magnetic properties of a non-Newtonian fluid on a porous surface

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Abstract:

In this paper, we have obtained the solution for the effect of an angle of inclination, mass flow rate, and skin friction on an MHD Non-Newtonian fluid of second-order type. The results are expressed in the terms of non-dimensional visco-elastic parameter (β) which is dependent on the frequency of excitation (σ) of the external disturbance and considering the angle of inclination (θ) magnetic parameter (m), and porosity (k) of the medium into account. We obtained expressions for velocity, skin friction and mass flow rate and compared with Newtonian.

Keywords:

Visco-elastic parameter, Angle of inclination, Frequency of excitation Magnification parameter, Magnetic parameter.

1. Introduction:

The study of fluid flow past porous boundaries has several applications in the fields of science, technology; engineering, biophysics, space dynamics, and astrophysics on the ablative surfaces, transpiration cooling of reentry vehicles and rocket boosters, and film vaporization in combustion chambers are a few such applications. In chemical and nuclear reactors, this problem has a greater significance. Over a period, in most chemical reactors, the slurry gets collected on the reactor walls. This results in the percolation of the chemical compounds through the boundaries causing either loss of production or consuming more reaction time. In some similar situations, to reduce the reaction time, which is a parameter of high importance, the reactor chamber is subjected to sinusoidal vibrations. Further, due to the presence of charged particles in the reactors, magnetic effects are induced. In this case, the problem becomes more complicated. Also, the problem has greater relevance, especially in biological systems where fluid secretion through glands is involved. Many times, in biological systems or chemical processing units, the secreted fluid is not only viscous but also elastico-viscous. The presence of the elastico-viscous nature of the fluid and the presence of a magnetic field causes drastic effects in evaluating the characteristic features of the fluid flow. This motivated the study and analysis of this problem in greater detail. The fluid flow through porous media occurs in ground-water irrigation, hydrology, drainage problems, and in absorption and filtration processes in chemical engineering. This subject has widespread applications to specific problems met in agriculture engineering and civil engineering, and many industries. Thus, the diffusion and flow of fluids through ceramic materials such as bricks and porous earthenware have long been a problem of the ceramic industry. The scientific treatment of the problem of soil erosion, irrigation, and tile drainage are present developments of porous media. In hydrology, the movement of trace pollutants in water systems can be studied with the knowledge of flow through porous media. The principles of this subject are useful in recovering water for drinking and irrigation purposes. The viscous flow over an oscillatory bottom earlier in his treatise on hydrodynamics was discussed by [1]. Then [2] studied the problem of two-dimensional steady-state Newtonian laminar flow in a channel with porous walls. An exact, analytical expression for the dependence of velocity on the pressure gradient has been derived. The response of a [3] second order visco- elastic fluid occupying a semi-infinite region due to harmonic oscillation of its bottom has been investigated later by [4-5]. The study of secondary flow in the rotating channel was conducted by [6]. The oscillatory motion of an electrically conducting visco-elastic fluid over a stretching sheet in a saturated porous medium was studied by [7]. A visco-elastic effect of non-Newtonian flow through porous media was studied by [8]. Then [9] studied the visco elastic effects in non-Newtonian steady flows through porous media. The flow of an elastic-viscous fluid over a stretching sheet was studied by [10]. Then [11] studied the flow of a visco-elastic fluid past a porous plate. Later [12] examined analytically the unsteady flow of Bingham fluid was caused by an abruptly applied pressure gradient. The flows of non-Newtonian fluids between two parallel porous walls, [13] obtained exact analytical solutions of the laminar flow of a second grade visco-elastic fluid employing two geometries. An oscillatory plate temperature effect of free convection flow of dissipative fluid between long vertical parallel plates was studied by [14]. After [15] presented an analysis of the transient free convective flow of a viscous incompressible fluid between two parallel vertical walls occurring because of asymmetric heating/cooling of the walls. Then [16] examined exact solutions for the incompressible viscous fluid of a porous rotating disk flow. The exact solution corresponds to viscous incompressible Newtonian conducting fluid flow due to a porous rotating disk [17]. The effect of suction and blowing on purely analytic solutions of the compressible boundary layer flow due to a porous rotating disk with heat transfer was studied by [18]. Later, [19] examined the problem of unsteady flow of an incompressible viscous electrically conducting fluid in the tube of elliptical cross-section under the influence of the magnetic field. Subsequently, [20] studied the unsteady flow of an incompressible viscous fluid in a tube of spherical cross-section on a porous boundary. Recently, [21] examined the problem of unsteady MHD flow of elastico – viscous incompressible fluid through a porous media between two parallel plates under the influence of a magnetic field.

The present paper is to study a class of exact solutions for the flow of incompressible electrically conducting elastic-viscous fluid of second-order fluid by considering the magnetic field, angle of inclination, and porosity factor of the bounding surfaces and compare the results with those in the Newtonian case. A uniform magnetic field of constant strength is supposed to be applied parallel to the y – direction. The induced magnetic field is negligible as compared with the applied magnetic field; the flow is laminar; it is valid for magnetic Reynolds numbers less than unity. Further, it is assumed that the magnetic Reynolds number is much less than unity, so the induced magnetic field is neglected in comparison with the applied magnetic field. We study the disturbance due to sinusoidal oscillation of the bottom of a semi-infinite depth. The results are expressed in terms of a non-dimensional porosity parameter k, which depends on the non-Newtonian coefficient φ_2 and the frequency of excitationσ. It is noticed that the flow properties are identical to those in the Newtonian case$\beta \to 0$, $k \to \infty$,$m \to 0$ and $\theta \to 0$.

2. Mathematical formulation of the problem:

The momentum equation of the fluid flowing through a generalized porous medium as suggested by [22] is given by.

$$\rho \frac{dq}{dt} = divS - \frac{\mu}{k}\bar{q}$$

and the continuity equation for incompressible homogeneous fluid

$$div\bar{q} = 0$$

Noll defined a simple material as a substance for which stress can be decided with the entire knowledge of the history of the strain. This is called simple fluid, if it has the property that all local states, with the same mass density, are intrinsically equal in response, with all observable differences in response being due to definite differences in history. For any given history $g(s)$, a retarded history $g_\psi(s)$ can be defined as:

$$g_\psi(s) = g(\psi s): 0 \le s \le \infty. 0 \le \psi \le 1 \qquad (1)$$

ψ being termed as a retardation factor. Assuming that the stress is more sensitive to recent deformation than to the deformations in the distant past, Coleman and Noll proved that the theory of simple fluids yields the theory of perfect fluids as $\psi \to 0$ and that of Newtonian Fluids as a correction (up to the order of ψ) to the theory of the perfect fluids. Neglecting all the terms of the order of higher than two in ψ, We have incompressible elastico viscous fluid of second order type whose constitutive relation is governed by:

$$S = -PI + \varphi_1 E^{(1)} + \varphi_2 E^{(2)} + \varphi_3 E^{(1)^2} \qquad (2)$$

Were,

$$E_{ij}^1 = U_{i,j} + U_{j,i} \qquad (3)$$

and

$$E_{ij}^2 = A_{i,j} + A_{j,i} + 2U_{m,i}U_{m,j} \qquad (4)$$

In the above equations, S is the stress tensor, U_i, and A_i are the components of velocity and acceleration in the direction of the i^{th} coordinate X_i while P is indeterminate hydrostatic pressure. The coefficientsφ_1, φ_2, and φ_3 are material constants. The constitutive relation for general Rivlin – Ericksen [23] fluid also reduces to equation (2) when the squares and higher orders of E^2 are neglected, while the coefficients are constants. Also, the non-Newtonian models considered by Reiner [24] could be obtained from equation (2) when $\varphi_2 = 0$ and naming φ_3 as the coefficient of cross viscosity. With reference to the Rivlin - Ericksen fluids $\varphi_2$may be called the coefficient of viscosity.

It has been reported that a solution of poly - iso - butylene in cetane behaves as a second-order fluid and that Markovitz determined the constantsφ_1, φ_2, andφ_3. In many chemical processing plants, slurry adheres to the reactor vessels and gets combined. As a result of this, the chemical compounds within the reactor vessel percolate through the boundaries causing a loss of production and consuming more reaction time. Given such technological and industrial importance wherein heat and mass transfer take place in the chemical industry, the problem of considering the permeability of the bounding surfaces in the reactors attracts the attention of several investigators.

Introducing the following non dimensional variables as:

$$U_i = \frac{\varphi_1 u_i}{\rho L} \qquad T = \frac{\rho L^2 t}{\varphi_1} \qquad \varphi_2 = \rho L^2 \beta \qquad P = \frac{\varphi_1^2 p}{\rho L^2} \frac{X_i}{L}{}_i = x_i \qquad \frac{Y_i}{L}{}_i = y_i$$

$$\varphi_3 = \rho L^2 \upsilon_c \quad A_i = \frac{\varphi_1^2 a_i}{\rho^2 L^3} \theta_0 = \frac{\theta \varphi_1}{L^2} S_{i,j} = \frac{\varphi_1^2 s_{i,j}}{\rho L^2} E_{i,j}^{(1)} = \frac{\varphi_1 e_{i,j}^{(1)}}{\rho L^2}$$

$$E_{i,j}^{(2)} = \frac{\varphi_1^2 e_{i,j}^{(2)}}{\rho^2 L^4} \quad K = \frac{kL^2}{\varphi_1} M = \frac{m\varphi_1}{L^2}$$

Where T is the (dimensional) time variable, and ρ the mass density, and L is a characteristic length.

We consider a class of plane flows given by the velocity components.

$$u_1 = u(y,t) \text{ And } u_2 = 0 \tag{5}$$

In the directions of rectangular Cartesian coordinates x and y. The velocity field given by (5) identically satisfies the incompressibility condition. The stress can now be obtained in the non-dimensional form as:

$$s_{xx} = -p + \upsilon_c (\frac{\partial u}{\partial y})^2 \tag{6}$$

$$s_{yy} = -p + (\upsilon_c + 2\beta)(\frac{\partial u}{\partial y})^2 \tag{7}$$

$$s_{xy} = \frac{\partial u}{\partial y} + \beta \frac{\partial}{\partial y} (\frac{\partial u}{\partial t}) \tag{8}$$

Given the above, the equations of motion in the present case of porous boundary will yield.

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \beta \frac{\partial}{\partial t}(\frac{\partial^2 u}{\partial y^2}) - (\frac{1}{k} + m + \theta)u \tag{9}$$

And

$$0 = -\frac{\partial p}{\partial y} + (2\beta + \upsilon_c)\frac{\partial}{\partial y}(\frac{\partial u}{\partial y})^2 \tag{10}$$

Eq (9) shows that $-\frac{\partial p}{\partial x}$ must be independent of space variables and hence may be taken as $\xi(t)$. Eq (10) now yields.

$$p = p_0(t) - \xi(t)x + (\upsilon_c + 2\beta)(\frac{\partial u}{\partial y})^2 \tag{11}$$

Considering$\xi(t) = 0$, the flow characterized by the velocity is given by:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \beta \frac{\partial}{\partial t}(\frac{\partial^2 u}{\partial y^2}) - (\frac{1}{k} + m + \theta)u \tag{12}$$

Where k is the non-dimensional porosity constant. It may be noted that the presence of β changes the order of the differential from two to three.

3. Disturbance of a liquid at rest due to the sinusoidal oscillations of the bottom.

The oscillations of a classical viscous liquid on the upper half of the plane $y \geq 0$ with the bottom oscillating with the velocity $\alpha e^{i\sigma t}$ are examined in the present case. The motion of the second-order fluid is governed by equation (12) with boundary conditions.

$$u(0,t) = \alpha e^{i\sigma t} \tag{13}$$

$$u(\infty,t) = 0 \tag{14}$$

Assuming the trial solution as:

$$u(y,t) = \alpha e^{i\sigma t} f(y) \tag{15}$$

$$f''(y) = p^2 f(y) \tag{16}$$

Where

$$p^2 = \frac{i\sigma + \frac{1}{k} + m + \theta}{1 + i\beta\sigma} = \frac{(\beta\sigma^2 + \frac{1}{k} + m + \theta) + i(\sigma - (\frac{\beta\sigma}{k} + m\beta\sigma + \theta\beta\sigma)}{(1 + \beta^2\sigma^2)} \tag{17}$$

The equation (17) is expressed in polar form.

$$p = r(\cos\cos(\frac{\pi}{4} - \frac{\varepsilon}{2}) + i \sin\sin(\frac{\pi}{4} - \frac{\varepsilon}{2})) \tag{18}$$

$r = \frac{[(\beta\sigma^2+\frac{1}{k}+m+\theta)^2+(\sigma-(\frac{\beta\sigma}{k}+m\beta\sigma+\theta\beta\sigma))^2]^{1/2}}{\sqrt{(1+\beta^2\sigma^2)}}$, $\varepsilon = (Q)$ and $Q = \frac{\frac{1}{k}+\beta\sigma^2+m+\theta}{\sigma-(\frac{\beta\sigma}{k}+m\beta\sigma+\theta\beta\sigma)}$

Also, the modified boundary conditions satisfied by $f(y)$are.

$$f(0) = 1, f(\infty) = 0 \tag{19}$$

Using equation (19) we get the solution for$f(y)$

$$f(y) = exp\ exp(-yr(cos\ cos(\frac{\pi}{4}-\frac{\varepsilon}{2}) + i\ sin\ sin(\frac{\pi}{4}-\frac{\varepsilon}{2}))) \tag{20}$$

And hence, we get the expression for the velocity field is given below.

$$u(y,t) = \alpha\ exp\ exp(i\sigma t - yr(cos\ cos(\frac{\pi}{4}-\frac{\varepsilon}{2}) + i\ sin\ sin(\frac{\pi}{4}-\frac{\varepsilon}{2}))) \tag{21}$$

The above flow is exemplified by a standing transverse wave with its amplitude rapidly deteriorating with increasing distance from the plane. This phenomenon is unrelated of $\upsilon_c$as saw for all two - dimensional flows.

Implying the amplitude of the disturbance (α), the magnification factor A^* of the amplitude of this wave is by

$$(A^*)^2 = (\text{RP of } u(y,t))^2 + (\text{IP of} u(y,t))^2 \tag{22}$$

$$A^* = \alpha\ exp\ exp(-y\sqrt{r}\ cos\ cos(\frac{\pi}{4}-\frac{\varepsilon}{2})) \tag{23}$$

Which is in the form of $A^* = e^{-\chi y^*}$

Were,

$$\chi y^* = \frac{y\sqrt{r}}{2}[cos\ cos\frac{\varepsilon}{2} + sin\ sin\frac{\varepsilon}{2}] \tag{24}$$

$$\chi = \frac{1}{(1+\beta^2\sigma^2)^{\frac{1}{4}}}\sqrt{\frac{Q+\sqrt{1+Q^2}}{1+Q^2}} \tag{25}$$

And

$$y^* = \frac{y(1+\beta^2\sigma^2)^{\frac{1}{4}}}{\sqrt{2}}[\frac{(\frac{1}{k}+m+\theta+\beta\sigma^2)^2+(\sigma-(\frac{\beta\sigma}{k}+m\beta\sigma+\theta\beta\sigma)^2}{(1+\beta^2\sigma^2)^2}]^{\frac{1}{4}} \tag{26}$$

The expression flow rate is$= \int_0^1 u(y,t)dy$ (27)

The skin friction on the lower plate is $= \left\{\frac{\partial u}{\partial y} + \beta\left[\frac{\partial^2 u}{\partial y\partial t} - \frac{\partial^2 u}{\partial y^2}\right]\right\}_{y=0}$ (28)

4. Conclusion:

In the present paper, we have obtained the exact solution of MHD elastic-viscous fluid flow with the angle of inclination and creating sinusoidal disturbances under the influence of a magnetic field on the porous boundary. As $m \to 0$ and $\theta \to 0$ the results obtained for the velocity field agree with that of [25] and [26] respectively. The case of Newtonian fluid can be realized as $\beta \to 0$, $k \to \infty$, $m \to 0$ and $\theta \to 0$.

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